XVIII. *Account of a Lithological Survey of Schehallien, made in order to determine the specific Gravity of the Rocks which compose that Mountain.* *By* John Playfair, *Esq. F. R. S.*

Read June 27, 1811.

The astronomical observations made on the mountain Schehallien, in 1774, were confessedly of great importance to science. They ascertained the power of mountains to produce a sensible disturbance in the direction of the plumb-line; of consequence, they proved the general diffusion of gravity through terrestrial substances, and afforded data for determining the medium density of the earth, compared with that of the bodies at its surface.

The skill with which this very delicate experiment was conducted by Dr. Maskelyne, and the ingenuity with which the results were deduced by Dr. Hutton, were worthy of the objects in view, and of the reputation which these distinguished men have acquired in their respective departments of the mathematical sciences.

One thing only seemed wanting to give to the determination of the earth's density all the accuracy that could be obtained from a single experiment, namely, a more precise knowledge of the specific gravity of the rock which composes the mountain, as being the object with which the mean density of the earth was immediately compared. The specific gravity of that rock was assumed to be to that of water as 5 to 2;

which, though it be nearly a medium when stones of every kind, from the lightest to the heaviest, are included, is certainly too small for Schehallien, the rocks of which belong to a class of a specific gravity considerably above the mean. The uncertainty arising from this source might not be of great amount, yet it was desirable that the quantity, or, at least, the limits of it should be accurately ascertained. In this light I knew, from repeated conversations, that the matter was regarded by both the gentlemen above named.

I had therefore long wished to attempt such a survey of the mountain as might afford a satisfactory solution of this difficulty; and having mentioned the circumstances to the Right Hon. Lord WEBB SEYMOUR, he entered readily into a scheme, which without the assistance of his skill and activity, I should have been quite unable to carry into execution.

Accordingly, in June 1801, we took up our residence in a small village as near as we could to the bottom of the mountain, and began our Mineralogical Survey, the result of which we think it our duty to submit to the Society, under the auspices of which the original experiment was undertaken.

It was obvious, that our first object must be to obtain specimens of all the varieties of rock in the mountain, which had any considerable difference in their external characters. These specimens must be such as had not been exposed to the action of the weather, were perfectly sound, with a fresh fracture, and taken from the living rock. In order to procure these, we soon found that it was not necessary to dig into the mountain or to blast the stones with gun-powder, for the native rock breaks out on the bare and rugged surface in abundance of places, and is so deeply intersected by the

streams that it was easy, by the assistance of the hammer only, to procure specimens having all the conditions requisite for our purpose.

Supposing, however, that all this was accomplished, it would be insufficient to determine the mean density of the mountain, unless the quantity of rock of each particular kind could also be estimated; at least nearly. It was necessary, therefore, to know what proportion of the mountain consisted of one species of rock, and what of another, without which the average could not be determined.

Had the mean density been the only thing wanted, it would have been sufficient to know the quantity of each variety of rock; but in the search we were engaged in, it was necessary to know not only the quantity, but the position of each of these varieties, relatively to the observatories on the south and north faces of the mountain. This will be evident, when it is considered that it was the effect of each portion of the rock on the plumb-line in these observatories that was the thing to be found, and that this effect must vary not only with the density of the rock, but with its distance from the observatory, and its obliquity in respect of the meridian. The mean density would therefore be insufficient for estimating the attraction of the mountain, could it be found ever so exactly; and it is easy to shew, that while the mean density of a heterogeneous mass, and also its magnitude and figure remain the same, its attractive force at a given point may be greatly changed by a different distribution of the materials it consists of, relatively to that point. In order then to form an estimate of this attraction we must know, at least nearly, these three things, the varieties of rock composing the mountain; the quantity

of each variety; and, lastly, the position of each relatively to the observatory. Fortunately the Geometrical Survey of the mountain, which had already determined not merely its superficial extent but its solidity, taken in combination, with some peculiarities in its structure, have enabled us to approximate, I hope with some tolerable exactness, to the knowledge of all these three circumstances.

The plan, then, which we proposed to follow, and which was necessary to be pursued if our Lithological Survey was to correspond in any degree to the accuracy of the Geometrical Survey, made under the direction of the Astronomer Royal, was to try to recognise the chain of stations which had been employed in that survey, in order that, by reference to those stations, we might be able to determine the points on the surface of the mountain from which our different specimens were collected. After these stations were discovered, we meant to traverse the mountain in various directions, and at any point where a specimen was taken, to determine our position by the bearings of any two of the stations that might be in sight, or by taking angles to three of them, or such other methods as occasion and circumstances might suggest. This was to be done where considerable variations in the external characters of the rocks gave reason to look for considerable variations of specific gravity. It was an operation that could not be necessary for every individual specimen, but it was one which must be necessary for determining the district over which stone of a particular character prevailed. In this part of the work we were to employ a theodolite, a sextant or a compass, according as more or less accuracy seemed requisite.

As the marks of the stations were all effaced except some

traces of the observatories (or rather the huts in which Dr. MASKELYNE had lived), and the two cairns on the top of the mountain, the discovery of the whole chain was a matter of some difficulty. By means, however, of the bearings, as given in Dr. HUTTON's paper, and the assistance of one of the guides who had been employed about the survey, we succeeded in finding out the stations; and as they were mostly on elevated points, we could distinguish them at a distance with sufficient exactness.

Schehallien belongs to one of the central ridges of the Grampians, which, stretching here from about SE. to NW. divides the vallies of the Tummel and the Tay. Though it be a part of this chain, it stands considerably separate from the rest on a base of a form somewhat oval, and having its figure distinctly defined by two streams that run, the one on the south, and the other on the north side of it. The lowest point in this base, which is on the NE. is 2467 feet below the summit of the mountain, and about 1094 above the level of the sea.

At the NW. extremity, Schehallien adheres to the main chain by means of a high ridge, depressed at its lowest point little more than 1500 feet under the summit. On the opposite sides of this neck the streams rise, which were before said to determine the base of the mountain; these streams, however, do not unite at the eastern extremity of the base, for there also a sort of neck, though very low in comparison of the former, connects Schehallien with the hills to the eastward.

Beyond the streams just mentioned, a range of inferior hills, some of them very low, springing from the main ridge

on the NW. encompasses the mountain, forming as it were a line of circumvallation round it, and on these were the stations which Mr. BURROWS, under Dr. MASKELYNE's direction, had chosen for the survey. Beyond these hills the ground falls down into a sort of plain of great extent on the north; on the south, less considerable and more uneven, yet such as to leave Schehallien very free and open in the direction of the meridian, and adapted by that means to shew the full amount of its action on the plummet. From the base its sides rise with a rapid, though unequal acclivity, and terminate not in a point, but in a ridge or narrow plane of a waving form, about a mile in length, and sloping regularly to the east, where it is 480 feet lower than at the western extremity. Though the sides are very rugged, they are less broken by deep ravines or bold projections than the other mountains of the same elevation in this quarter of the Grampians; for, beside the high neck which has been already mentioned as uniting Schehallien to the mountains on the west it has only one other saliant ridge, which runs out to the NE. and overlooks the plain with a very steep and precipitous aspect. In some directions, and when viewed from a considerable distance, the harsh features of the mountain are wonderfully softened; it acquires a very beautiful conoidal shape, and from thence derives the name by which it is known among the inhabitants of the low country.

The rock of Schehallien, like that of all the mountains in its vicinity, is of the class called primitive; and is disposed for most part in great parallel plates, or strata, nearly vertical, stretching from SE. to NW. They are indeed so nearly vertical, that a deviation of 15° from the perpendicular is rarely

to be met with, except toward the base of the mountain, where it is sometimes greater, and is subject to considerable inequalities. The strata on the north side of the mountain lean a little toward the north, and those on the south toward the south. All these variations, however, are inconsiderable, and in general the strata may be set down as nearly vertical.

But though in their disposition all the rocks of Schehallien agree pretty nearly, they differ considerably in their mineralogical characters. A large proportion of the mountain, and that which constitutes the most elevated part, is formed of a granular quartz, extremely hard, compact, and homogeneous. The whole mass from about the level of the two observatories up to the summit of the mountain, is of this stone. Lower down, again, on every side, the rock is a schistus containing much mica and hornblend; and the division into parallel and vertical plates is more obvious than in the granular quartz. This last, however, is sometimes found in the lower parts, forming thin, vertical plates, interstratified with the hornblend and mica slate, and all together preserving their parallelism with a neatness and accuracy which a work of art could hardly exceed. This is particularly to be observed in the bed of the *burn* of Glenmore, the stream that defines the base of the mountain on the south, and which toward the lower part of its course intersects the strata to a great depth.

Besides these two kinds of rock, we meet in several places toward the base of the mountain with a granular and micaceous limestone highly cristallized, which in one or two places ascends to a considerable height. All these rocks are disposed in strata; but there are also veins or dykes of porphyry and greenstein, which traverse the mountain in different directions;

one of the former kind, of great breadth, cuts it right across from NE. to SW. not far from the point of its greatest elevation. There is no where any appearance of metallic veins.

The quartzy rock of Schehallien is extremely hard and homogeneous, and by its slow and uniform decay, has no doubt given rise to that massive shape, and comparatively unbroken surface, which have been already remarked. Yet even here the work of time is abundantly evident; for the rock being much cut by fissures transverse to its stratification, it separates and falls down in large prismatic fragments. Some of these are of a vast size, and being extremely durable, the accumulation of them where the ground is not too steep to permit them to lie, is very great, so that large tracts of the sides of the mountain are covered with cubical blocks of granular quartz, resting on one another, and *steadied* only by their own weight.

It is remarkable of this quartzy stone, that when exposed for some time to the weather, it acquires the lustre and appearance of white enamel, so that the old weather beaten surface is more clear and shining than that which is immediately produced from a fresh fracture. The reason seems to be, that the stone does not consist of pure quartz, but along with the grains of quartz has a great number of grains of felspar interspersed, which when it is first broken give it an opaque and earthy appearance. These are soon dissolved by the action of the weather; and there is then left over the surface a coat of pure quartz, which has the semi-transparency and vitreous gloss belonging to enamel.

The felspar which enters into the composition of the rock here described, is not always in grains, but in some specimens

is found regularly crystallized. The crystals however are small and very thinly disseminated; were they in more abundance this stone might be accounted a granite, as Professor JAMESON has remarked of a stone of the same kind which he found in the island of Jura.

From the vertical position of the strata we may infer with some probability, that the rock which breaks out any where at the surface, continues the same through the interior of the mountain, in the direction of a perpendicular plane, down to its base, or perhaps to an indefinite depth. The same stratum usually remains of the same nature to a great extent, whenever we have an opportunity of examining it, whether in a horizontal or a perpendicular direction; and it is not to be doubted that the same holds when no such opportunity occurs. When, therefore, we have on the surface a bed of mica slate, or of granular limestone, or of granular quartz, the probability is that the whole stratum all through the mountain is composed of the same materials. I must however confess, that I do not think that this probability is as strong with respect to granular quartz, as it is with respect to the micaceous rocks. These last compose the great mass of the Grampians; and their characters, though not every where the same, change very slowly, and pass from one to another by imperceptible gradations. To the granular quartz this rule does not equally apply; it is not general among the mountains of this tract; it sometimes breaks off suddenly, and is replaced by rocks of a very different nature. We cannot therefore with the same confidence assume the existence of this rock in intermediate points, when we only see it in the extremes. This much, however, we know with certainty, that the whole of the upper part of the

mountain, from about the level of the south observatory to the summit, consists of granular quartz, as no other stone is to be met with any where in that tract. This is the part above O, in the section of the mountain; and the only question is, whether we shall consider the part in the interior of the mountain, immediately under this mass, as consisting of the same rock. When we first examined Schehallien, Lord. WEBB SEYMOUR and I were both of opinion that this was the most probable supposition. Since that time, however, having had an opportunity of examining some other of the Grampians where granular quartz is found at the summit, and where, nevertheless, it is certain that the same rock does not go down into the interior, there has appeared some reason to suspect that this may be true of Schehallien. As the result of the calculus with regard to the earth's density is materially affected by these suppositions, I have given the result as I had first deduced it on the hypothesis, that the interior of the mountain is of granular quartz; and also on the hypothesis that the quartz is confined to the upper part; and that the lower part is entirely composed of mica and hornblend slate.

In the computation which Dr. HUTTON made of the attraction of Schehallien, he supposed its mass divided into 960 vertical columns, and he computed the force with which each of these columns disturbed the direction of a plummet suspended in either observatory, supposing them all homogeneous and two and a half times as dense as water.* Now knowing from our survey, and the combination of geometrical with mineralogical observations, the specific gravity of each of these columns at the surface, and conceiving (what we have shown, with one

* Phil. Trans. Vol. LXVIII. (1778), p. 689, &c.

exception, to be probable) that the column remains the same through its whole length, we can compare the real attraction with that assigned to it in Dr. Hutton's calculation. The attraction of any column computed on his hypothesis being divided by 2.5, and multiplied by the true specific gravity, will give the real attraction, or effect in disturbing the plumb-line. It is on this principle that our correction is formed, though simplifications occurred that very much diminished the labour of the computation, the nature of the rock leading us in the end to distinguish only two differences of specific gravity; and the ingenious deductions of Dr. Hutton, together with the excellent order that prevails in his computation, having made it easy to follow a route which he had cleared of all its greatest difficulties.

However, as it was impossible to determine before hand how much the specific gravity of these rocks might differ, it was necessary to conduct the survey so that every individual column, had it been necessary, might have had its specific gravity defined. For this purpose the mountain was traversed in various directions, and the points at which a transition was made from rocks of one character to those of another were carefully noted, and their position ascertained. In selecting the specimens which were to represent the rocks of the several districts into which the mountain thus became divided, attention was paid both to the prevailing stone, and to that which was least common, in order that we might, if possible, get possession both of the mean and the extremes. This was in general the principle that guided our choice of specimens, but the application of it in detail to particular instances does not admit of being explained. The reasons in every such

case that determined us to take one stone and reject another, could only be perceived by an observer on the spot, whose eye was accustomed to judge of the varieties, the plenty or the rarity of the minerals that passed in review before him, by indications which it is impossible to describe in words. We are here therefore with reluctance compelled to request from the reader more credit than we are able to prove to him that we deserve. We know that in doing this we are craving an indulgence which no wise and candid observer ever wished to possess; we sincerely regret that the nature of the subject forces us to make this demand, and that the part of our work which it was most difficult to perform to our own satisfaction, is quite incapable of being explained to the satisfaction of others.

Catalogue of Specimens from Schehallien.

The rocks of the mountain may be divided, as already remarked, into three classes; granular quartz; mica, and hornblend slate; granular limestone. The specific gravities were ascertained by the late Dr. KENNEDY, and it is therefore unnecessary to add that their accuracy may be perfectly relied on. The pieces weighed were between 1000 and 4000 grains: most commonly between 2000 and 3000. Different pieces of the same specimen were often examined. The water used was distilled, and always of a temperature between 60 and 61 degrees.

Quartz.

1. Gray sandstone containing mica in thin layers. Specific gravity = 2.6435.

2. White quartz, very pure. Fracture vitreous. Occurs in

beds chiefly on the NE. side of the mountain. Specific gravity = 2.6437.

3. Quartzy sandstone of a whitish gray colour with thin layers of mica. Specific gravity = 2.6296.

4. Quartzy sandstone. White colour with layers of mica. Much indurated. Somewhat ferruginous. Specific gravity = 2.65367.

5. Indurated sandstone with spiculæ of mica interspersed. Specific gravity = 2.6460.

6. Sandstone much indurated, vitreous shine, interspersed with mica. Specific gravity = 2.6269.

7. Granular quartz from near the summit; contains grains of felspar. Specific gravity = 2.6274.

8. Granular quartz; nearly the same with the preceding. Specific gravity = 2.6109.

9. Sandstone fine grained, slightly marked with iron veins. Specific gravity = 2.6296.

10. Sandstone fine grained, more indurated than the preceding. Specific gravity = 2.6576.

11. Sandstone containing calcareous matter. Specific gravity = 2.6656.

12. Granular quartz, very compact and indurated, but of a stratified structure; a little mica in thin plates. From a mean of several. Specific gravity = 2.6452.

13. Granular quartz of a flesh colour; imperfect crystals of felspar thinly disseminated. This specimen from near the top. From a mean of several peices. Specific gravity = 2.6387.

The mean of these thirteen specimens gives 2.6398 for the upper or quartzy part of the mountain.

Mica and Hornblend Slate.

1. Hornblend slate very compact. Specific gravity = 3.0642.

2. Micaceous schist, with hornblend and a small mixture of quartz. Specific gravity = 2.9385.

3. Black micaceous schistus, fine grained containing hornblend. Specific gravity = 3.0476.

4. Micaceous schistus containing pyrites and quartz in fine grains. Specific gravity = 2.7293.

5. Micaceous schistus tinged with an oxid of iron. Specific gravity = 2.7935.

6. Micaceous schist with thin plates of mica and hornblend transverse to the stratification. Specific gravity = 2.7907.

7. Micaceous schist with quartz in small grains. Specific gravity = 2.7499.

8. Another specimen nearly the same. Specific gravity = 2.7728.

9. Compact micaceous schistus, grains of felspar and quartz intermixed. Specific gravity = 2.71845.

10. Nearly the same with the preceding. Specific ra vity = 2.7206.

The medium specific gravity of these ten specimens is 2.83255.

Limestone.

1. Granular limestone of a gray colour, containing some mica. Specific gravity = 2.7087.

2. Granular limestone, silver coloured, stratified structure. Specific gravity = 2.8890.

3. The same, bluish, highly crystallised. Specific gravity = 2.76057.

4. The same, finer grained, containing thin layers of mica. Specific gravity = 2.7419.

5. The same, gray coloured, and the crystals larger. Specific gravity = 2.7302.

The mean specific gravity of these five specimens is = 2.76607.

Gran. Quartz.		Mic. and Calc. Schist.		
Numbers.	Specific Gravities.	Specific Gravities.	Numbers.	
1	2.6435	3.0642	1	Mic.
2	2.6437	2.9385	2	Mic.
3	2.6296	3.0476	3	Mic.
4	2.65367	2.7293	4	Mic.
5	2.6460	2.7935	5	Mic.
6	2.6269	2.7907	6	Mic.
7	2.6274	2.7499	7	Mic.
8	2.6109	2.7728	8	Mic.
9	2.6296	2.71845	9	Mic.
10	2.6576	2.7206	10	Mic.
11	2.6656	2.7087	1	Calc.
12	2.6452	2.8890	2	Calc.
13	2.6387	2.76057	3	Calc.
Mean	2.639876	2.7419	4	Calc.
		2.7302	5	Calc.
		2.81039	Mean	

From the inspection of the preceding table, it is evident that the specimens relatively to their specific gravity may be divided into two classes sufficiently distinct from one another. The specimens of granular quartz are in specific gravity comprehended between 2.61 and 2.66, nearly, and the mean is 2.639876. The micaceous rocks, including the calcareous, are contained between the limit 2.7 and 3.06, the mean of all the 15 specimens being 2.81039. Now it happens fortunately, that these two classes of rocks distinguished by their specific gravity are also distinguished by their position, so that the line which separates them can be accurately traced out on the face of the mountain. As to the arrangement of the same two classes of rock in the interior of the mountain, there are only two different suppositions, as already observed, which possess any degree of probability, and the result of each is hereafter to be given. The curve line in the plan of the mountain divides the quartzy from the micaceous rocks.

I shall now proceed to state the principles on which the present investigation is founded, and the result to which it has led.

According to Dr. HUTTON's construction, if O (Fig. 1.) be the place of the plummet in the south observatory, ON the direction of the meridian, and if with a radius ON = 13333 feet, or $\frac{40000}{3}$, a quadrant of a circle be described, viz. WRN; if ON be divided into 20 equal parts, and if from O as a centre, through each of these points of division, circles be described: lastly, if through O, radii as OH, OG, &c. be drawn such that the sine of the angle which each of them makes with the meridian shall differ from the sine of that

which the contiguous radius makes with the meridian by $\frac{1}{12}$ of the radius; that is, if sine GON — sine HON. $= \frac{1}{12}$, &c. then shall every one of the twenty concentric rings be divided into twelve spaces, upon each of which if columns of homogeneous matter be supposed to stand, and to be of such altitudes as to subtend equal angles from O, the attraction of each column on the plummet at O, in the direction of the meridian ON, will be the same.

The attraction of any of these columns, as of that which stands on the base GHKL, is measured thus. Let $b =$ GL, the breadth of the column in the direction of the radius, $= \frac{40000}{3.12} = 666.666$ feet; $d =$ difference between the sines of the angles of azimuth, or sin. GON — sin. HON $= d$; E $=$ angle of elevation of the column above O: then the attraction $= b\,d \times$ sin. E.*

I have also used a theorem in these computations, which gives an accurate value of the attraction of a half cylinder of any altitude a, and any radius r, on a point in the centre of its base, and in the direction of a line bisecting the base. Let A be equal to that attraction; then A $= 2\,a$ Log. $\frac{r + \sqrt{a^2 + r^2}}{a}$, or A $= 2\,a$ Log. $\frac{r}{a}\left(1 + \sqrt{1 + \frac{a^2}{r^2}}\right)$.

Fig. 2. represents a vertical section of Schehallien in the direction of the meridian of the south observatory O.† The line QR represents the level of the lowest part of the base of

* Phil. Trans. Vol. LXVIII. p. 751.

† The observatories O and P are not in the same meridian; they are however nearly so; and the section through P in the direction of the meridian would not differ sensibly from that which is here given

the mountain. P is the north observatory; the part of the section coloured with a reddish brown represents the granular quartz, supposed here to constitute the interior as well as the summit of the mountain. The dark colour represents the schistus; the two belts of grey are the limestone strata on the north and south sides. OR or the elevation of the south observatory above the lowest part of the base of the mountain is 1440 feet.

Fig. 3. is a section of the mountain in the direction perpendicular to the meridian of the south observatory. This section, though not referred to in any of the computations, is useful for enabling one to form an idea of the structure and figure of the mountain.

Draw OL (Fig. 2.) parallel to the horizon. With OR as an axis, and with a radius of 13333 feet, suppose a cylinder to be described, and let it be cut into two semi-cylinders by a plane passing through O R perpendicular to RQ the meridian. Then the whole of the mountain on the north side of this plane disturbs the direction of the plummet by drawing it toward the north. But the part of the mountain to the north of this plane, and between the levels O and R is equal to one of the semi-cylinders above mentioned, *minus* the empty space between the surface of the ground and the horizontal plane passing through O. If therefore S denote the attraction of the semi-cylinder, and V that which the void space would have were each pillar in it to consist of matter of the same density with the part of the same pillar which is under the surface, S—V will represent the attraction or disturbing force of all that part of the mountain which is north of OR and under the level of O.

Again, putting S′ and V′ to express the same things for the part of the mountain to the south of O, the whole attraction of that part equal $S' - V'$, and this acting in an opposite direction to the other, or tending to restore the plummet to its mean position, is to be subtracted from the former quantity, so that the whole disturbing force by which the part of the mountain below the level of O acts upon the plummet at O, is $S - V - S' + V'$. To this the attraction of the upper part of the mountain, or that which is above the level of O, being, as it happens, wholly to the north is to be added, and if it be called T, the whole disturbance on the plummet at O is $S - S' - V + V' + T$.

In Dr. HUTTON's computation, S and S′, or the attraction of the half cylinders on opposite sides of O are equal to one another, the cylinder being supposed to consist of matter of the same density throughout; they must therefore destroy one another, and consequently, according to that hypothesis, they did not require to be calculated. The case here is not the same; for the matter in the two semi-cylinders not being of uniform density, nor having its inequalities similarly distributed, the attraction of each must be calculated in order that their difference, or $S - S'$ may be found.

If Σ, Σ', U, U′, and T′ denote the same quantities for he observatory P on the north side of the mountain, then the disturbing force on the plummet at P, $= \Sigma - \Sigma' - U + U' + T'$; and so the whole force which alters the direction of gravity is $S - S - V + V' + T + \Sigma - \Sigma' - U + U' + T'$.

The computation of these quantities for the columns in the quadrant north-west of O, will serve to explain the method followed in all the rest.

The whole cylinder of which OR is the axis being divided into 960 columns, the quarter of it must consist of 240, all of which, as far as their bases are concerned, are of equal force in attracting the plummet at O, so that the difference of their effects depends entirely on their altitude. Let O, (Fig. 1.) represent the south observatory, ON the meridian, and the quadrant ONW a horizontal section through O of one-fourth of the cylinder, on which the bases of the columns are marked as in the figure. Let *a b c* be the bounding line of the quartz projected on the plane of this section, the columns whose bases are within that line being supposed wholly of quartz, and those without it of micaceous schistus. If we suppose the columns that have their tops in this section to be extended downwards to the depth of 1440 feet, we shall have the quarter cylinder divided into 240 columns, that would be of equal disturbing forces, were they of equal density, and equal apparent depression below the point O. The inspection of the figure serves to distinguish the columns of quartz from those of micaceous schistus. In those columns which consist of both rocks, the proportion of the quartzy to the micaceous part could be judged of with sufficient accuracy by the eye. To assist the eye, however, the figure being first constructed to a large scale, I used to stretch a fine thread either in the direction of a radius passing through O, or in a line at right angles to that direction (according as the case seemed to require), so as to divide the quadrilateral into two quadrilaterals equal, as nearly as the eye could judge, to the irregular divisions made by the boundary of the quartz and schistus. The proportion of the parts was then easily ascertained. Now, by the first of the theorems laid down above, the attraction of

any column on the plummet at O, estimated in the direction of the meridian ON, if b be the breadth of it in the direction of the radius, d the difference of the sines of the azimuths of the two edges, E the angle which the length of the column subtends at O, if its density were $= 1$, would be $bd \times \sin. E$. But if the density of the rock be expressed by any other number, the attraction just found must be multiplied by that number in order to give A the real attraction of the column. Thus if Q denote the density of the granular quartz, and M that of the micaceous schistus, we have in the former case $A = bd\,Q. \sin. E$, and in the latter, $A = bd\,M. \sin. E$. In these formulas $b = 666.66$ feet, and $d = \frac{1}{12}$, by the construction already explained; therefore $A = (55.55)\,Q \sin. E$, or $= (55.55)\,M. \sin. E$.

The calculation of sin. E is very easy, for the length of each column or its depth below O being 1440 feet, and the middle of the first ring being 333.33 feet distant from O; of the second 1000, reckoning from O, if n be the number of any ring, the distance of its centre from O is $\frac{2n-1}{3} \times 1000$, so that $\tan. E = \frac{1440}{\frac{2n-1}{3} \times 1000} = \frac{3 \times 1.44}{2n-1}$.

The sine corresponding to this tangent taken from the tables, and multiplied into 55.55, and the product into Q or M, will give the attraction of the column. Therefore to have the attraction of the ring of columns of the order n, the quantity now obtained must be multiplied by 12, that being the number of columns in one ring, having all by hypothesis the same altitude, so that the whole attraction of the ring $=$ $(666.66)\,Q \sin. E$, &c.

The attraction of each of the twenty rings being thus computed, their sum gives the attraction of the quarter cylinder.*

From the projection of the columns in Fig. 1. it appears that the first six rings in the NW. quadrant are entirely of quartz, that the five following are mixed, being partly quartzy partly micaceous, and that the nine remaining columns are wholly micaceous. The little table that follows contains the proportions of quartzy and micaceous rock in the five rings just mentioned.

Rings. \ Sectors.	1	2	3	4	5	6	7	8	9	10	11	12
7	$\frac{7}{10}q$ $\frac{3}{10}m$	$\frac{8}{10}q$ $\frac{2}{10}m$	$\frac{9}{10}q$ $\frac{1}{10}m$	q	q	q	q	q	q	q	q	q
8	m	m	m	$\frac{1}{8}q$ $\frac{7}{8}m$	$\frac{2}{5}q$ $\frac{3}{5}m$	$\frac{8}{9}q$ $\frac{1}{9}m$	q	q	q	q	q	q
9	m	m	m	m	m	m	$\frac{2}{5}q$ $\frac{3}{5}m$	q	q	q	q	$\frac{7}{11}q$ $\frac{4}{11}m$
10	m	m	m	m	m	m	m	$\frac{1}{8}q$ $\frac{7}{8}m$	$\frac{7}{8}q$ $\frac{1}{8}m$	q	q	$\frac{1}{4}q$ $\frac{3}{4}m$
11	m	m	m	m	m	m	m	m	m	$\frac{1}{4}q$ $\frac{3}{4}m$	$\frac{1}{4}q$ $\frac{3}{4}m$	m

This table is constructed only for that part of the north-west

* It was most convenient to compute the attraction of the quarter cylinder in this way, though merely an approximation, because the columns of which it consisted are not all of the same specific gravity. In the case of their being homogeneous, the attraction of the quarter cylinder might be computed *exactly* by the second theorems given above. Indeed I investigated that theorem for the purpose of examining into the degree of accuracy that this approximation actually possessed; and I had the satisfaction to find, that when the two methods were applied to the same half or quarter cylinder, (supposed homogeneous,) the difference of the results did not exceed a two thousandth part of the whole. This demonstrated in a very satisfactory manner the accuracy of the method pursued by Dr. HUTTON.

quadrant in which columns occur of two different rocks; and a rectangular cell is assigned to each column in the five rings to which the table refers. The letters Q and M denote quartz and mica; and where one letter only occurs, the column is entirely of the rock which it denotes. In the cells where both letters occur, the column consists of both rocks in the proportion expressed by the fraction prefixed to each letter. Thus in the seventh ring, the first quadrilateral is $\frac{7}{10}$ quartz and $\frac{3}{10}$ mica; the second, $\frac{8}{10}$ quartz and $\frac{2}{10}$ mica; the third, $\frac{9}{10}$ quartz and $\frac{1}{10}$ mica; the remaining nine being entirely quartz.

Now to apply the tables thus constructed to the computation of the attraction of any of the quarter cylinders, it must be observed, that sin. E is to be found for any column in the way already explained, and is then to be multiplied by $b\,d$, b being $= \frac{2000}{3}$ and $d = \frac{1}{12}$, so that $b\,d = \frac{2000}{3 \times 12} = \frac{500}{9}$, and therefore the coefficient of Q or M is $\frac{500}{9} \times$ sin. E.

When the whole ring is of the same rock, the coefficient of sin. E computed for a single column is to be multiplied by 12, so that the whole attraction of the ring $= \frac{500}{9} \times 12 = \frac{2000}{3} = 666.66$, as before determined.

In the mixed columns the sine of E is to be multiplied both into $b\,d$, and into the fraction prefixed to Q for the quartz, and to M for the mica; or if we would include the whole ring, as E is the same for all the columns contained in it, we must multiply $b\,d$ by the numbers denoting the proportion of quartz or of mica in the whole of that ring. Thus in ring 5, the first in the preceding table, the whole quartz $= 11.4$, the

sine of E being $=.3157$, and the attraction due to the quartz $= \frac{11400 \times Q}{3 \times 6} \times .3157 = (129.9423)\,Q.$

In this manner the attraction of the whole cylinder on the plummet at O is readily computed; but it muct be diminished on account of the part by which this cylinder rises above the surface of the ground. The quantity that is to be subtracted is computed from the sines of the depressions of the tops of the different columns below the observatory O; and Dr. HUTTON's paper either actually exhibits* those sines, or furnishes us with the means of readily computing them.† When a ring is wholly of the same species of rock, the sum of the sines of the depressions of all the columns in that ring is given in the tables, and needs only to be multiplied by $\frac{500}{9}$ to give the coefficient of Q or M as far as that ring is concerned.

Again, when in the same ring some columns are of quartz and others of mica, the sines of depression must be computed trigonometrically for each column by help of the data contained in the tables above referred to. The sum of those sines for the quartz columns being multiplied by $b\,d$ gives the coefficient of Q.

Where the same column is of two different kinds of rock, the sine of the depression, or of E, must be multiplied into $b\,d$, and divided in the proportion of the numbers prefixed to Q and M in the cell belonging to the column.

All this may be illustrated by the calculation of the attraction of the columns belonging to the above table. In the seventh ring the first columns are mixed, the next three are

* Phil. Trans. Vol. LXVIII. from p. 769 to 776.

† Ibid. from p. 759 to 765.

entirely of quartz, and the remaining six are wanting, that is to say, their tops are not depressed below the level of O, as may be seen in Table V. of Dr. HUTTON'S paper. From that table it also appears, that the depth of the summit of the first column of the seventh ring below the level of O is 250 feet; of the second 240, of the third 200, of the fourth 150, of the fifth 60, and of the sixth 30. From these measures the angles of depression may be computed. Thus, if 250 be divided by the radius of this ring, viz. $\frac{13000}{3}$, we have .0577 for the tangent of the depression, or of E, and the sine which corresponds is .0568. As $\frac{7}{10}$ of this column consists of quartz, we must take $\frac{7}{10}$ of this sine for the proportional part of the coefficient of Q. In like manner, the sine of the depression of the top of the second column is .0545, of which taking $\frac{8}{10}$, we get .0436 for the part of the coefficient of Q belonging to this column. So also for the third ring, the proportional part of the sine is .04149. The fourth, fifth, and sixth columns being entirely of quartz, no proportional parts are to be taken; their sines, computed as before, are .0346, .0138, .0069; and the sum of all these six numbers is .18015.

Calculating in the same way for all the columns that are entirely or partly of quartz in the north-west quadrant, we have the amount of the whole = .2534. Now the total sum of the sines of the depressions in this quarter, is 13.534. (See Dr. HUTTON'S computations, page 83). From this number, if .2534 be taken away, there will remain 13.2806 as the coefficient of M, arising from the depressions of the micaceous columns.

Now the sum of the sines belonging to the quartz in the

quarter cylinder itself has been found = 53.3532, from which taking away .2534, there remains .53.0998 for the entire sum of the sines belonging to the quartz under the level of O in the north-west quarter of the mountain.

In like manner the sum of the sines computed for the micaceous columns and parts of columns in the north-west quarter cylinder, is 23.0124, from which taking away 13.2806, the deficient or negative part, there remains 9.7318. The numbers thus found being multiplied by $\frac{500}{9}$, give the coefficients of Q and M for the north-west quarter of the mountain below the level of O, and make its attraction = (2949.99) Q + (540.655) M.

A similar computation being made for the quarter cylinder on the north-east of O, we have its attraction = (2974.299) Q + (577.98) M, to which adding their former attraction (2949.99) Q + (540.655) M, we have S—V, or the attraction of the mountain on the north side of O, and under the level of O = (5924.289) Q + (1118.635) M. In like manner the attraction of the south-west quadrant deduced partly from the quarter cylinder, and partly from Dr. HUTTON's calculations = (1049.18) Q + (1819.66) M, and of the south-east = (1567.394) Q + (1052.129) M; the sum of which, or S′—V′, gives (2616.574) Q + (2871.789) M; to be subtracted from the former, in order that we may have the total disturbing force of the part of the mountain below O, which is therefore = (3307.715) Q — (1753.154) M.

Lastly T, or the attraction of the part of the mountain above O (which is on the north), when computed from the sums of the sines of the elevation of the columns above O, as given

by Dr. HUTTON, is found = (2474.389) Q + (150.855) M, which, added to the preceding, gives the whole attraction on the plummet at O = (5782.104) Q — (1903.209) M.

The same quantities calculated for P, the north observatory, are (8061.022) Q — (3127.05) M. To which adding the attraction just found for O, we have (13843.126) Q — (5030.214) M = the total force of attraction increasing the convergency of the plumb line on opposite sides of the mountain.

Now if D be the mean density of the globe, it follows from Dr. HUTTON'S calculations that 87522720 × D is the measure of the attraction of the whole earth. But the Astronomer Royal having found by his observations, that the sum of the deviations of the plumb line on opposite sides of the mountain is 11.6 seconds, the attraction of the earth is therefore to the sum of the opposite attractions of Schehallien, as radius to the tangent of 11″.6, that is as 1 to .000056239, or as 17781 to 1; or, making an allowance for the centrifugal force arising from the earth's rotation, as 17804 to 1. Therefore 17804 : 1 : : 87522720 × D : (13843.126) Q — (5030.214) M, so that $\frac{87522720}{17804}$ D = (13843.126) Q — (5030.214) M, and hence D = $\frac{13843126 \times Q - 5030214 \times M}{4915902}$, or D = (2.816) Q — (1.023) M.

If we suppose Q = 2.639876 and M = 2.81039, as in the table above, D = 4.55886.

Dr. HUTTON makes D = $\frac{17804}{9933}$ multiplied into 2.5, the supposed density of the rock,* which gives D = 4.481, considerably less than the preceding. If in the formula D = (2.816) Q — (1.023) M we make Q = M = 2.5, the result should

* Phil. Trans. Vol. LXVIII. p. 781.

agree with Dr. HUTTON's, and does so very nearly, making $D = 4.482$.

In all this, we have proceeded on the supposition that the granular quartz not only constitutes the summit of the mountain, or the part above the level of the observatories, but that it also descends into the interior of the mountain down to its base, where it is bounded by the curve line *a b e* (Fig. 1). On the other supposition mentioned above, that the granular quartz does not constitute the interior nucleus of the mountain, but is confined to the upper part of it, the rest consisting of micaceous schistus, our formula, after undergoing certain changes, may also be accommodated to this hypothesis. In the value of the attraction of the part of the mountain below O, viz. $(3307.715)\,Q - (1753.154)\,M$, we must suppose $Q = M$, when the above quantity becomes $(1554.561)\,M$. To this we are to add T, or the attraction of the part of the mountain above O, which remains the same as before, viz. $(2474.389)\,Q - (150.055)\,M$, to which if we add $(1554.561)\,M$, the sum $(2474.389)\,Q + (1404.506)\,M$ is the whole attraction on the plummet at O, according to this new hypothesis.

If the same changes are made with respect to the observatory P, we shall have the total attraction. Now the attraction of the part of the mountain below $P = (5593.347)\,Q - (3172.15)\,M$, which if $Q = M$ becomes $(2421.197)\,M$. Also the attraction of the part above P is $(2467.675)\,Q + (45.15)\,M$. If to this be added $(2421.197)\,M$ the amount, or $(2467.675)\,Q + (2466.347)\,M$ is the total attraction on the plummet at P.

To the total attraction at $O = (2474.389)\,Q + (1404.506)$

M, add the total attraction at P $= (2467.675)$ Q + (2466.347) M; then the total attraction by which the direction of gravity is altered by the mountain, is (4942.064) Q + (3870.853) M. Hence as before $D = \frac{(4942.064)\,Q + (3870.853)\,M.}{4915.902.}$ or $D = (1.0053)\,Q + (0.78743)\,M$.

Here if we make as before $Q = 2.639876$ and $M = 2.81039$, we shall have $D = 4.866997$. This therefore is the mean density of the earth, on the supposition that the interior of Schehallien, on a lower level than the observatories, consists of micaceous schistus. The measure thus obtained, for the mean density or mean specific gravity of the earth, is above that of any of the precious stones, and is nearly a mean between the results of Dr. HUTTON and Mr. CAVENDISH. According to the former, $D = 4.481$; according to the latter, $D = 5.48$, the mean of which is 4.98. The difference between this and the last of our results is nearly $= .1$, or less than a forty-fifth part.

If we are to consider the experiments on Schehallien singly, it seems highly probable that the mean density of the earth is contained between the limits deduced from the two different suppositions concerning the structure of the mountain, so that it cannot be less than 4.5588, nor greater than 4.867. The mean of these is nearly 4.713.

It is however desirable that an element so important in physical astronomy, as the mean density of the earth, should be the result of many experiments. The principle on which those at Schehallien were made seems the most likely to lead to accurate conclusions. In the selection of the places fit for such observations, the homogeniety of the rock is a condition

that merits particular attention, and is hardly to be looked for any where but among granite mountains, as they alone afford a perfect security that their interior and exterior are composed of the same materials. Granite is the lowest of the rocks, and whenever it appears at the surface we may be assured, that on penetrating deeper, we shall meet with no other.

It is therefore to the primitive mountains, and among them to the granitic, that such experiments as those made at Schehallien ought to be confined. The want of homogeneity will then be on the outside of the mountain only, and can easily be estimated. The granite may be covered at the bottom of the mountain and even to a considerable height on its sides with beds of gneiss, mica slate, hornblende slate, &c., the quantity and position of which can easily be ascertained by observation.